

N.B. (i) All questions are compulsory.

(ii) Attempt ANY THREE-sub questions out of FOUR in each question.

(iii) Figures to the right indicate marks.

Q.1. (a) Explain the concept of probability density function of a continuous random variable X. State any two properties of probability density function. 7

(b) In a continuous distribution, the density function is 7

$$f(x) = kx(2-x) \quad 0 \leq x \leq 2$$

$$= 0 \quad \text{otherwise}$$

Find k, mean and variance of the distribution.

(c) Find the cumulative distribution function F(x) for the random variable X with p.d.f. as 6

$$f(x) = x \quad 0 < x < 1$$

$$= 2 - x \quad 1 \leq x \leq 2$$

$$= 0 \quad \text{otherwise}$$

Draw a sketch of f(x) and F(x).

(d) The life in hours (X) of a certain electronic component is a continuous random variable with probability density function 6

$$f(x) = \frac{150}{x^2} \quad x \geq 150$$

$$= 0 \quad \text{otherwise}$$

Determine the form of cumulative distribution function of X. Hence obtain the probability that an electronic component would last for at most 250 hours.

Q.2. (a) If X follows Rectangular distribution in (a,b), write down p.d.f. of X. Hence obtain expressions for mean and variance of X. 7

(b) A study of divorced men showed that the interval of time (in years,x) between the day of their marriage and the day of their divorce has the following p.d.f. 7

$$f(x) = 0.2 e^{-0.2x} \quad x > 0$$

$$= 0 \quad \text{otherwise}$$

Find the probability that one Mr.X who got divorced during the last year spent (i) at most one year (ii) at least 5 years of marital life before the divorce.

- (c) The income distribution of a group of 10000 persons was found to be Normal with mean Rs. 750/- p.m. and standard deviation Rs. 50/- p.m. What percentage of this group had income (i) exceeding Rs. 668/- (ii) exceeding Rs. 832/- (Given : Area under standard normal curve between ordinates at  $\pm 1.64$  is 0.8990) 6
- (d) The probability of a defective item,  $p$  is 0.10. If a sample of 400 items is drawn from a very large lot, find the probability that there are (i) 30 or more defectives (ii) Exactly 30 defectives. 6
- Q.3. (a) Explain the concept of estimation. What is (i) statistic (ii) parameter (iii) estimator? 7
- (b) The mean life of a large lot of fluorescent tubes is 1570 hrs. with a standard deviation of 150 hrs. A sample of 100 tubes is drawn from it with replacement. Find the probability that mean life of these tubes will (i) exceed 1600 hrs. (ii) not exceed 1540 hrs. (iii) lie between 1550 hrs. and 1600 hrs. 7
- (c) If  $p$  denotes the probability of a fuse working properly, the following procedure is adopted to test the hypothesis  $H_0 : p = 0.9$  against the alternative  $H_1 : p = 0.8$ . Inspect four and retain  $H_0$  if all are working properly. Calculate the probabilities of Type I and Type II errors. At what level of significance is the test based? 6
- (d) A group of 121 boys obtained mean intelligence quotient (I.Q.) of 84 while a group of 81 girls obtained 80. If the s.d. of I.Q. is given to be 10, can we conclude that there is a significant difference between their performances? Use 5% level of significance. 6
- Q.4. (a) Define for a continuous random variable its Cumulative distribution function. State any two properties. 5
- (b) The demand of cakes (in kg.) at a bakery shows Rectangular distribution in (1000,1500). Find the probability that on a certain day the demand would be (i) at least 1200 kg. (ii) between 1100 kg. and 1450 kg. 5
- (c) If the marks in a particular subject are assumed to follow Normal distribution with mean 40 and variance 9, find how many out of 1000 students get marks between 43 and 46. 5
- (d) An item is packed in lots of 100 each. Let  $m$  denote the mean number of defectives in a packet of 100. To test  $H_0 : m = 2$  against  $H_1 : m = 3$ , it is decided to select one packet and inspect the items in it. If it contains four or more defectives, it is proposed to reject  $H_0$ . Find the level of significance for the test. 5

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